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LETTER TO THE EDITOR

A new solution of Ernst's equation

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Abstract. A three-parameter solution of Ernst's equation, unfortunately not asymptotically flat, is presented. For special values of the parameters, the solution reduces to the Curzon solution.

In the elegant formulation of the problem of stationary gravitational fields by Ernst (1968, 1974) the field equations reduce to

$$\mathrm{Re}(\mathscr{E})\nabla^2\mathscr{E} = \nabla\mathscr{E} \, . \, \nabla\mathscr{E}.$$

A survey of known solutions of this equation was given by Kinnersley (1975). Since then generalisations of the Tomimatsu–Sato (TS) solutions have been found by Yamazaki (1977a, b) and Cosgrove (1977). Asymptotically non-flat solutions corresponding to contractions of the TS metrics have been obtained by Ernst (1977).

The solution I have found is given in the usual spherical coordinates, r, θ as follows:

$$\mathscr{E} = \frac{A + \mathrm{i}B}{N},$$

where

$$N = p^2 + q^2 r^4 \sin^4\theta \exp(2m/r)$$

$$\begin{aligned} A = {} & q^4 r^8 \sin^8\theta \exp(3m/r) + p^2 r^2 \sin^2\theta \exp(m/r)[2r^2q^2(1 - 8\cos^2\theta) \\ & + 8qr\cos\theta(2mq\cos\theta - a) + 4qm\cos\theta - 4q^2m^2\cos^2\theta - a^2] \\ & + p^4\exp(-m/r) \end{aligned}$$

and

$$\begin{aligned} B = {} & \exp(2m/r)\{16pq^3\sin^2\theta\cos^2\theta\, r^4 + 8pq^2r^3\sin^2\theta\cos\theta(a - 2q\cos\theta) \\ & + pqr^2[-4q^2m\cos^4\theta + \cos^2\theta(4qm + 6p^2 - a^2) \\ & - 4pqm\cos\theta\sin^2\theta + a^2 - 2q^2] \\ & + 2q^2r[a\cos\theta - pqm(3\cos^2\theta - 1)] + 2q^2m\cos\theta(pqm - a)\} \\ & + 2p^3qr^2(3\cos^2\theta - 1) + 2p^2r[a\cos\theta - pqm(3\cos^2\theta - 1)] \\ & + 2p^2m\cos\theta(pqm - a). \end{aligned}$$

In this solution m and a are arbitrary real parameters, while p and q are related by $p^2 + q^2 = 1$.

For $q=0$, $a=0$, B vanishes and A reduces to the familiar expression for the Curzon metric.

As A/N and B/N behave like r^4 and r^0, respectively, for large r, the solution cannot be asymptotically flat. The dominator N vanishes nowhere. One may thus suspect that the solution is free of singularities.

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